# THE MATHEMATICS TEACHER

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## INTRODUCTORY COURSE IN MATHEMATICS.

By David Eugene Smith.

In spite of all that has been said in this country in opposition to mathematics in the past few years, the feeling of certainty still exists in the intellectual world that the science is not dead, is not dying, and is not stagnant; that it touches more lines of human interest to-day than ever before; and that its values have only been accentuated by the efforts made to relegate it to the position of formal grammar, formal rhetoric, and formal logic. Indeed, it may safely be said that mathematics stands more firmly to-day than ever before, not only in the minds of what is commonly called the intellectual class but in the opinions of the man in the shop and of the man who has so recently been in the trenches on the battlefields of France.

There are various reasons for this improvement of the position of mathematics. Some of these reasons are concerned with the character of certain criticisms which have been advanced, and with some of the strange, not to say wild, suggestions which have been made for improvement in the teaching of the subject. The man of real scholarship and the man of intellectual leadership is certain to look upon many of the criticisms as puerile, and the teacher who has made a success in his profession will as surely look upon many of the suggestions for improvement as the strange vagaries of men who are wholly ignorant of the problem. In particular, the criticism that algebra has nothing that the average citizen needs is one that has

materially strengthened the position of the subject among thoughtful students of education, while the suggestion that all mathematics should be blended without reference to the radical differences of purpose of certain branches, or that it should be taught only by some project method, has made the successful teacher suspicious of the aid to be expected from the professor of education.

As a matter of fact, the discussion has brought to the front the values of a subject like algebra; it has shown more clearly the distinctive purposes of branches like algebra and geometry; and it has shown that there is no more reason for making mathematics merely an adjunct to the work of the shop, which is often the essence of the project method, than there is for abolishing physical education except as it is an adjunct to manual work on the farm or in the factory.

Teachers of mathematics have been accused of standing back of the shield of tradition, and the same may be said of the church and of our most valued social customs; they have been charged with failing to state with definiteness the precise purposes of their courses, and the same may very likely be said of every course in education in every school of education in this country; they have been said to desire the use of public funds for that which is not practical, and the same may be said of every course in music, the fine arts, or *belles lettres* everywhere.

But out of all this dispute, often running into the ridiculous or the pitiful, there has come considerable good. I am inclined to think that mathematics is rated higher throughout the country, and that the science of education, or at least some portion of its retainers, has been rated lower. This I infer from careful reports covering a wide range of our territory.

The teachers of mathematics have undoubtedly been spurred to greater activity, and this is generally a good thing. They have been urged to examine their problem more thoroughly, to encourage the elimination of those features which have lost their essential character, to compare our work with the work done in other countries, to see how the subject can be made more real to the pupil, to determine what new features can be introduced to replace those which are eliminated, and to do all this by adding positive values which no fair mind would question.

As a result of this searching of the soul of our science and of our own souls as teachers there has come one decided change in our work, and for this we have reason to thank the school administrators. I refer to the movement that would extend the high school down to the seventh school year. It is immaterial what name we give to the school which begins in the seventh grade, whether we speak of the Junior High School as covering grades seven, eight, and nine, or of the "six and six plan," or of departmental teaching in the grades; the essential thing is that we have in this movement the possibility of introducing mathematics in a more rational way than the one heretofore followed by us, and to embody in our work the best that the rest of the world has to offer in the teaching of the subject. Of all the advanced countries of Europe and America, and we may also include Japan, we have been unquestionably the most backward in our introductory work in mathematics; but at last the opportunity has come for us to profit by the experience of other nations and by the results of our own investigations into the needs of our people.

The bases upon which we should rest our new structure are unquestionably the needs of our people in the home, in various industries, and in commercial and other activities. Whatever of disciplinary values mathematics has,—and it is interesting to see the diversity of view of those who so vainly seek to prove that it has none,—this disciplinary value will be developed as effectively from mathematics resting upon those foundations as from a science which, in its first stages of presentation, is based solely upon considerations of abstract logic.

If we recognize, as most of the world does, that the utilitarian operations of arithmetic have been essentially covered at the end of the sixth school year, we still have to teach those general applications which everyone needs to know but which the child's experiences do not enable him to grasp in the earlier years. At the same time we have to make sure that the child's powers of computation do not atrophy through lack of use, a thing that is sure to happen if arithmetic is entirely stopped at that time.

If we have three years of a Junior High School, therefore, we have this general proposition to consider: What shall be the

extent, the nature, and the position of arithmetic in order that its general applications shall best be understood and the habit of accurate computation retained and strengthened?

Without answering this question at present, we may consider the position of geometry and algebra. A few years ago there was some local agitation in the country in favor of fusing, as it was called, the work in algebra with that in geometry. Failing in this, there was the effort to make demonstrative geometry a branch of applied mathematics, or at least to seek applications for its various propositions. Neither of these movements succeeded, and for very sound psychological reasons. In the first place, the aims and methods of demonstrative geometry are as distinct from those of algebra as are the aims and methods of chemistry or physics. There are a few points of contact, and various interesting analogies, but the demonstration of a proposition in geometry is undertaken for an entirely different purpose and by an entirely different method from those found in the solution of a linear equation, even though the latter has its analogue in the science of form. Mathematics is a traditional name; it does not happen to cover music but it does cover the art of computation, although in the Greek civilization music was included and the art of computation was not. Simply because geometry and algebra are both called mathematics, by present fashion, is no reason why two such essentially different subjects should be forced into a most unnatural wedlock.

Furthermore, the idea of searching for a series of practical applications for every proposition of demonstrative geometry is to lose entirely our clearness of vision. Let it be distinctly affirmed for the comfort of the educational iconoclast,—asking due pardon for resorting to the *argumentum ad populum,*—let it be distinctly said that demonstrative geometry is not, never was, and never will be taught because of its immediate practical applications. No workman in a shop ever applies any proposition in geometry that he could not apply just as well, if it were told to him, without proving it. He knows the Pythagorean Theorem before he studies geometry, he knows how to use it, and he also knows the fundamental property of similar figures. If the immediate application of the proofs of the propositions of demonstrative geometry to money making is the purpose of

the science, then demonstrative geometry must *ipso facto* cease to exist. It is on this account that the search for applications of any considerable number of demonstrations has been abortive,—and the demonstration, not the nearly obvious geometric fact, is the essence of geometry as we commonly understand the terms. One might as properly seek for a practical application of every painting in an art gallery, of every symphony of an orchestra, of every beauty in nature, and of every noble thought that stirs the soul.

Just as the word "arithmetic" has entirely changed its meaning in the last four centuries, however, and just as the name "algebra" has assumed a new significance within the same period, and just as the word "geometry" as completely changed its meaning after the time of Thales, so we are meeting with desirable changes to-day. To have students begin their work in demonstrative geometry of the older type is to lead a certain non-intellectual group into a hopeless maze. To have them begin their algebra on the plan of twenty years ago is equally unreasonable. No one believes in this method at the present time for the mixed class of students who enter our high schools, and all teachers of any promise earnestly wish to break away from it. We have happily reached a time when we can extend our concept of geometry and allow it to cover the intuitive as well as the demonstrative phase of the subject. Similarly, we have reached a time when we are safely extending our ideas of algebra to include a line of practical and interesting applications which render the introduction to the science far more psychological than anything that was known a generation ago.

And finally, by the way of introduction, let it be said that we have come to realize that algebra and geometry do not constitute all that there is of secondary mathematics. To realize what a rich field we have, it is only necessary to look into the work of countries like France or England and see how naturally a simple trigonometry fits into the early courses.

With these preliminary observations, I venture to suggest certain principles which should, I believe, guide us in planning the introductory work in mathematics.

First, arithmetic should occupy the pupil's attention in the first half of the seventh grade, so that the break in mathematics

shall not seem too pronounced. It should not be mere drill in computation, however; it should consist of those lines of application for which he was not prepared in his earlier work, and should be based upon real social needs,—his relation to the home, the store, and industry, and including the need for keeping accounts and avoiding waste.

This being done, the second half year may best be given to intuitive geometry. We have unconsciously recognized this, in a small way, by putting some mensuration into our arithmetic in the later years of our elementary school. The subject is, however, more extensive than that. The human mind naturally approaches geometry with three needs to be satisfied. Given an object, we may ask three questions of a geometric nature: (1) What is its shape? (2) How large is it? (3) Where is it? There are no others, for questions of value, color, odor, or use are not geometric in nature. Here, then, is the domain of intuitive geometry, a domain whose bounds are fixed by psychology, but one whose broad extent includes a large number of fields of immediate practical application. After these questions have been answered, there still remains the question of proving the correctness of the statements, and this is the domain of demonstrative geometry which the student may enter a year or so later.

In this domain of intuitive geometry the student learns the immediately practical side,—how to describe the shapes of objects, how to measure any objects that he is likely to need to measure in ordinary life, and how to locate points on a pattern, in the field, on a map, or in space, for each of which the number of genuine applications is very great. In all the work in measurements the student meets the formula in a natural way, he comes to know its meaning and its value, and thus he makes a beginning in algebra before the name has come to have any significance to him.

The next step is also determined by psychological considerations. The pupil has already seen some use for algebra, he has applied simple formulas in his intuitive geometry, and he is now ready to enter the new field. In making this entrance psychology again directs the way by telling us to continue to build upon the formula as we have already begun to do in Grade VII.

Since the pupil has a background of applied arithmetic and of mensuration, we should use the accumulated material as a basis for further formulas, thus correlating the geometry and algebra in the most effective manner. The student has now a reason for manipulating these formulas for the purpose of deriving his own rules, and so he comes to the equation. This, however, is not the equation of the older school, interesting as that puzzle was to most students and profitably as it may therefore be used as drill material; it is now the real equation, introduced naturally and used with a definite purpose. Following this work there naturally comes the graph of the formula, and thus the graph becomes something more than a mere picture of statistics of population of a territory or of the attendance in a school. If the graph is a curve, this curve may drop below zero, and hence the negative quantity enters naturally and tangibly. In a half year, therefore, the student comes to know the nature of and the uses for the four great features of elementary algebra,—the formula, the equation, the graph, and the negative number. He may know nothing of algebraic addition, or of any other operation; he may not know what a polynomial is; but he knows some of the great things of algebra, and he has accumulated a few practical tools which he can use in his reading about the simpler laws of mechanics and industry.

What does psychology now suggest? Naturally that the student should at once apply his algebra to arithmetic as well as to mensuration. To meet this natural demand there may now be introduced those wider ranges of business arithmetic for which the student's maturity of experience in mathematics and in his relation to life has prepared him. Such topics as the arithmetic of trade, of industry, of the bank, of corporate business, of daily life, of relations to the State, and of thrift and investments may well make use of the tools which algebra has furnished.

The student will thus come to know the chief uses of intuitive geometry and of the first part of elementary algebra. If he leaves school at the end of Grade VIII, he takes with him the immediately practical facts that he will need in his general reading and in such special walks of life as he is likely to enter.

If, however, the student proceeds in his school course,—then

what? It seems reasonable to give him some chance of knowing what real mathematics is like. We cannot yet tell what this new field will mean to him. He may have succeeded in the intuitive stage and fail here; the teacher may have failed to arouse his interest there, while here he may find that approach to exact truth which may stimulate him to worthy achievements. It seems criminal, therefore, to close to him the opportunity of trying himself in the larger domain; in other words, a subject which touches such a wide range of human interests and which offers the only knowledge of deductive logic that the school has as its command, should be made known to every student. This means the requiring of mathematics in the ninth school year.

What should be the nature of this mathematics? While intuitive geometry comes before algebra, being the more tangible, concrete subject, demonstrative geometry belongs late in such a course as this for the reason that it requires considerable maturity of judgment. We may therefore say that, in general, the first steps in the post-intuitive mathematics should be (1) algebra, (2) trigonometry, (3) demonstrative geometry. The algebra may properly include the fundamental operations, justified by constant reference to arithmetic, a subject upon which it can and should pour a flood of light; simple equations with practical applications; and an introduction to quadratics, with applications to those formulas likely to be met in the reading of books and papers about such subjects as automobiles, airplanes, and domestic science. Whether the student can factor an expression, or handle an awkward fraction is of little moment at this stage; the important thing is that he shall see what algebra means and shall be encouraged to continue the study if he has any gift in this direction.

After this introduction to the science of algebra the student should come to know the meaning of trigonometry, and he should learn how to use a tangent or a sine for practical purposes. Trigonometry does not naturally follow demonstrative geometry; it naturally follows intuitive geometry and algebra. It is chiefly an algebraic science based upon geometric intuition.

Finally, as an introduction to mathematics, it is the right and privilege of every student to know what demonstrative geometry means. It is here that many students first awaken to the

significance of mathematics. The appreciation of a demonstration and the contact with exact truth,—the value of these experiences in the adolescent period cannot be overestimated, and the educator who would bar a student from the possibilities of such experiences assumes a heavier burden than I should wish to carry..

It may be said that the student will not know much mathematics after such a course. This is true, and it is equally true that he does not know much mathematics after he completes the ninth school year at present. What he will know, however, is what three important parts of mathematics are about; he will know certain very important uses of the subject; and he will have tried himself out. If it is said that this smattering will dull the edge of interest, the answer is that every student who enters the École normale supérieure or the École polytechnique of France has gone through a similar smattering, and yet no schools in the world stand higher in mathematics. The teachers of mathematics have nothing to fear from such an introduction; the ones who have cause to fear are those who would eliminate from our high school courses mathematics as a serious subject.

This, to my mind, constitutes the proper approach to mathematics. It is also, to my mind, the minimum course to be offered and the maximum amount to be demanded. The student has now weighed his mental powers in the balance; he has shown whether or not he may profitably proceed with a subject which may carry him into the purest realm of thought, into the most profound speculations, or into the most important applications of science. Thereafter his work in mathematics should be the result of careful, sympathetic conference with his advisers at home and in the school. He may well stop at this point, or he may proceed farther, and the school should set no limit upon the extent of his progress provided there is sufficient demand to make the formation of classes justifiable. The educator who says that we, the teachers of mathematics, must show the value of mathematical astronomy, for example, should first show us the value of their own courses on the one hand, or of art, music, and the love of one's fellows on the other hand.

What I ask for mathematics I would ask for every branch of knowledge that touches such a wide range of human interests,—

art, science, language, *belles lettres,* history;—that every student in our high schools shall come to know what these subjects mean, how they touch and have touched humanity, and whether or not he or she is fitted to and cares to enter upon the serious study of one or more of them. In this way we shall improve our teaching, we shall do our duty to the youth of our country, and we shall stimulate the real study of a science in which we have a faith that does not fail us.

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## A MESSAGE TO COLLEGE STUDENTS.

Do not get caught in the receding tide of the great war. Set yourselves at once to look forward. Remember that the world must be built up again, and it looks as if there was an opportunity to make the world better than it has ever been before. We believe there is a chance of preventing this thing from ever happening again, of building up mankind to something nearer a perfect condition, where every man can use his own faculties to the utmost, which, after all, is the great pleasure in life; where every man who has a heart and an ambition will be able to develop himself for something worth doing. Remember that, and look forward, you fellows that are young. Do not look back into the receding wave, but look forward into the crest that is coming on ahead of you. As in this war, so in civil life—your own right hand will teach you terrible things if you will only make your own right hand strong and use it for the right purpose, and begin now at once.—*President A. Lawrence Lowell, Harvard University.*